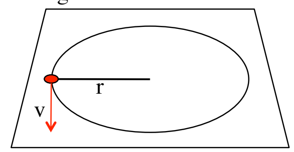
Problem 6.1

$m = 3.00$ kg



Until now, when a sole force has been exerted on a body along the line of the body's motion, the resulting acceleration has changed the body's velocity vector in the sense that the magnitude of the velocity has changed—the body has sped up or slowed down. We are about to look at a different class of acceleration problems. Known as centripetal force and/or centripetal acceleration problems, these are associated with forces and accelerations that are applied PERPENDICULAR to the direction of motion so as to change the DIRECTION of the velocity vector. That is, these forces don't speed up or slow down a body, they push a body *out of straight-line motion*.

Centripetal acceleration, which is always in a *radial direction* relative to the arc the body is traveling upon, is numerically proportional to the square of the body's velocity magnitude "v" and inversely related to the radius of the arc "R" being traced (this will be proved in class). That is:

$$a_{cenripetal} = \frac{v^2}{R}.$$

With this preamble, what are the range of speeds the object can experience if the string will break when the tension is $mg = (25.0 \text{ kg})(9.80 \text{ m/s}^2) = 245 \text{ N}$?

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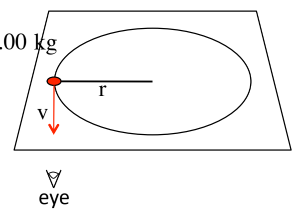
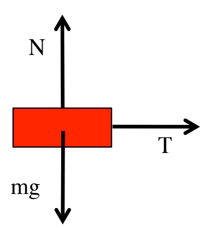


Although this is a problem you could easily do without much thought, there is no reason not to deal with it from a Formal N.S.L. approach. That's what we will do.

Starting with a f.b.d.:

We need an orientation that allows us to see ALL THE PERTINENT FORCES ACTING. In this case, the orientation that allows us to see the normal, gravitational and tension forces is one that looks straight at the puck as it comes toward us (see the eye on the sketch). Doing that yields the f.b.d. shown to the right.



The next step used to seem silly, primarily because in the context of acceleration that made bodies pick up speed or slow down, it was. Here, though, it is not.

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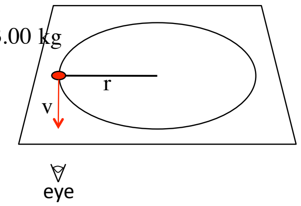
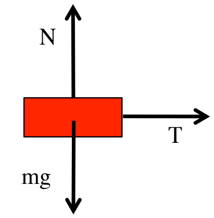
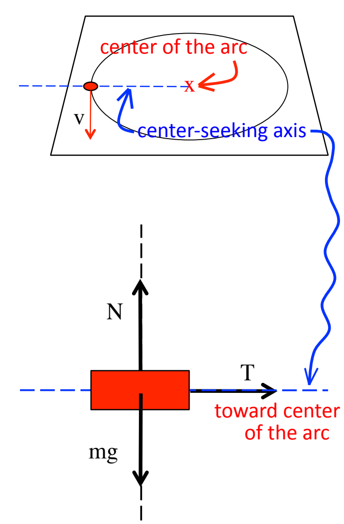


The command is: Determine the line of acceleration for the system, and place an axis along that line.

IF YOU DON'T DO THIS RIGHT, EVERYTHING YOU DO FROM HERE ON OUT WILL BE WRONG. Why? Because if you set up the coordinate axes wrong, you will sum the forces along a "wrong axis," then try to put that sum equal to "$m\left(\frac{v^2}{R}\right)$," and your expression will be nonsense!



To determine the "center seeking direction," identify the center of the arc upon which the body is moving. Once identified, draw an axis *from the body through that center*. That will be the centripetal direction. Then draw an axis perpendicular to that. This is all shown on the sketch.



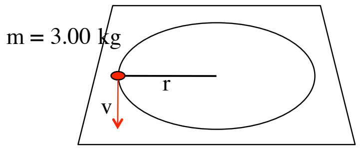
With the centripetal direction identified, and as there are no off-axis forces to be broken into components, we can use N.S.L. along that axis writing:

$$\underline{\sum F_{centripetal}:}$$

$$T = ma_c$$

$$= m\left(\frac{v^2}{R}\right)$$

So what do we know? At low speeds, the tension is low and there is no fear of breaking the string. It is only at 245 newtons that a break will occur. The velocity that does that is:



$$T = m\left(\frac{v^2}{R}\right)$$

$$\Rightarrow \quad (245\ \text{N}) = (3.00\ \text{kg})\frac{v^2}{(.800\ \text{m})}$$

$$\Rightarrow \quad v = 8.08\ \text{m/s}$$

The body can travel anywhere up to 8.08 m/s before the string breaks.